

# Modification in Ampere's Law & Displacement Vector :-

According to the Ampere's law we know that 'In a particular loop (closed surface) the line integral of magnetic field is equal to the  $\mu_0$  times of the total current flowing in the loop enclosed by the surface i.e;

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\text{or } \int B \cdot dl = \mu_0 I \quad \text{--- (1)}$$

From the definition of current density,

$$J = \frac{dI}{ds}$$

$$\text{or } dI = J ds$$

on integrating  $I = \int J ds$  --- (2)

put in (1)

$$\int B \cdot dl = \mu_0 \int J ds \quad \text{--- (3)}$$

on applying stoke's theorem in I.H.S of above eq<sup>n</sup>, we get

$$\int \text{curl } B \cdot ds = \mu_0 \int J \cdot ds$$

$$\int (\text{curl } B - \mu_0 J) \cdot ds = 0$$

if here  $\int ds \neq 0$ , then bracket term should be equal to zero.

$$\text{curl } B - \mu_0 J = 0$$

$$\text{curl } B = \mu_0 J$$

$$\text{curl } H = J \quad \left[ \because B = \mu_0 H \right] \quad \text{--- (4)}$$

taking divergence on both sides, we get

$$\text{Div } (\text{curl } H) = \text{Div } J$$

From vector calculus the Div. curl of a vector quantity is always equal to zero.

Then from above eq<sup>n</sup>, we get

$$\boxed{\text{Div } J = 0} \quad \text{--- (5)}$$

This is known as Ampere's law for electrostatics.

From eq<sup>n</sup> of continuity

$$\text{div } J + \frac{d\rho}{dt} = 0$$

$$\frac{d\phi}{dt} = 0$$

which is only possible in electrostatics.

Hence, a modification was made in Ampere's law which gives the exact result of current flowing in a particular loop.

For this a term  $J_d$  is added in Ampere's law which is known as Displacement Vector (Displacement current density).

Then the Ampere's law converts in the form of :-

$$\text{Curl } H = J + J_d \quad \text{--- (6)}$$

Now taking div. on both sides

$$\text{Div Curl } H = \text{Div } J + \text{Div } J_d$$

$$\text{Div } J + \text{Div } J_d = 0 \quad \text{--- (7)}$$

This modification in Ampere's law is based upon the Maxwell's law and is also known as Maxwell's Modification.

From Maxwell's eq<sup>n</sup> we know that

$$\text{Div } \vec{D} = \rho \quad \text{--- (8)}$$

where  $D$  is electric displacement vector and whose value is given by:  $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$   ~~$\epsilon_0 \vec{E}$~~

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad \text{--- (9)}$$

→ Polarisation

put the values of eqn (8) and (9) in eqn (A)

$$\text{Div } \vec{J} + \frac{d}{dt} (\text{Div } \vec{D}) = 0$$

or

$$\text{Div } \vec{J} + \frac{d}{dt} \text{Div} (\epsilon_0 \vec{E} + \vec{P}) = 0$$

on comparing with eqn (7), we get

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

$$\therefore \text{Div } \vec{P} = 0$$

$$\vec{E} = \frac{\partial \vec{E}}{\partial t} = \vec{J}_d$$

put in eqn (6)

$$\text{curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\text{curl } \vec{H} = \vec{J} + \frac{\partial \vec{E}}{\partial t}$$